Solutions of Equations of One Variable

Scientific Computing 372

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Bisection Method

An interval $[a_{n+1}, b_{n+1}]$ containing an approximation to a root of $f(x) = 0$ is constructed from an interval $[a_n, b_n]$ containing the root by first letting

$$p_n = a_n + \frac{b_n - a_n}{2}.$$

Then set

$$a_{n+1} = a_n \quad \text{and} \quad b_{n+1} = p_n \quad \text{if} \quad f(a_n)f(p_n) < 0,$$

and

$$a_{n+1} = p_n \quad \text{and} \quad b_{n+1} = b_n \quad \text{otherwise.}$$

Stopping criteria:

(1) A midpoint coincides with the root.

(2) Length of search interval is less than prescribed tolerance.

(3) Number of iterations exceeds some preset bound.

Secant Method

The approximation p_{n+1}, for $n > 1$, to a root of $f(x) = 0$ is computed from the approximations p_n and p_{n-1} using the equation

$$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}.$$

Stopping criteria:

(1) When $|p_n - p_{n-1}|$ is within a given tolerance.

(2) Number of iterations exceeds some preset bound.

Important Note

The Secant method does not have the “root-bracketing” property of the Bisection method, and consequently, it does not always converge. However, when it does, it is usually much faster than the Secant method.

Newton's Method

The approximation p_{n+1} to a root of $f(x) = 0$ is computed from the approximation p_n using the equation

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}.$$

Stopping criteria:

(1) When $|p_n - p_{n-1}|$ is within a given tolerance.

(2) Number of iterations exceeds some preset bound.

Important note

If f' is continuous, Newton's method is satisfactory, provided that $f'(p) \neq 0$ and that a sufficiently accurate initial approximation is used. The condition $f'(p) \neq 0$ is not trivial: It is true precisely when p is a simple root, i.e., if a function q exists with the property that, for $x \neq p$,

$$f(x) = (x - p)q(x), \quad \text{where} \quad \lim_{x \to p} q(x) \neq 0.$$

If p not simple, Newton's method may converge, but not necessarily rapidly.