Mathematics knowledge

1. (a) What is the value of $2^{10}$? \[1\]

(b) What is the decimal value of the base-2 number $101010_2$? \[1\]

(c) What is the value of $5! + 1$? \[1\]

(d) What is the value of $\binom{7}{5}$? \[1\]

(e) What is the value of $77 + 76 + 75 + \cdots + 3 + 2 + 1$? \[1\]

2. (a) Calculate $\frac{d}{dx} x \log x^2$. \[2\]

(b) Calculate $\int x^5 + \sin x \, dx$. \[2\]
3. Calculate the following matrix product:

\[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 1 \\
3 & 1 & 2
\end{bmatrix}
\begin{bmatrix}
2 & 0 & 1 \\
0 & 1 & 2 \\
1 & 2 & 0
\end{bmatrix}
\]

4. In a specific lottery, 6 numbers are drawn (without replacement) from the range 1...50.

(a) Calculate the probability of correctly guessing the 6 numbers if order matters. [2]

(b) Calculate the probability of correctly guessing the 6 numbers if order does not matter. [2]

5. Consider the following sequence of numbers:

1 7 3 6 14 3 −1 1 2

(a) What is the mean value of the sequence? [1]

(b) What is the median value of the sequence? [1]

(c) What is the mode of the sequence? [1]

(d) What is the standard deviation of the sequence? [1]
6. Simplify the expression

\[(a \Rightarrow b) \Rightarrow (\neg b \Rightarrow c)\]

Computer Science knowledge

7. (a) How many bits to a byte (usually)?

(b) According to the IEEE standard, a 32-bit, single-precision, floating-point number \(N\) is defined to be

\[N = (-1)^S \times 1.F \times 2^{E-127}\]

where \(S\) is the sign bit, \(F\) the fractional mantissa, and \(E\) the biased exponent. A floating-point number is stored as \(S : E : F\), where \(S\), \(E\), and \(F\) are stored in 1 bit, 8 bits, and 23 bits, respectively. What is the decimal value of the floating-point number \(C1E00000\) (hexadecimal notation)?

(A) 26    (B) 15    (C) –26    (D) –28    (E) 59

8. Name three sorting algorithms and, for each algorithm, give its worst-case asymptotic time consumption. Example: XYZ sort \(O(n)\).
9. Explain or define each of the following concepts:

(a) Dynamic memory

(b) Binary search tree data structure

(c) Heap data structure

(d) Graph

(e) Object-oriented programming

(f) SQL

(g) TCP/IP

(h) UML
(i) Critical section

(j) Context switching

(k) Coupling and cohesion

(l) BNF (Backus-Naur form)

(m) Factory design pattern

(n) Contrapositive

(o) Garbage collection

(p) Iterator
10. (a) What does the abbreviation “NP” (as in “P v. NP”) stand for? [1]

(b) What does it mean when we say that a problem is $NP$-complete? [2]

(c) Name one $NP$-complete problem. [1]

11. (a) What are the components of a finite automaton? [2]

(b) What is the difference between a DFA (deterministic finite automaton) and an NFA (nondeterministic finite automaton)? [2]

(c) What is the relationship between regular languages, DFA’s, and NFA’s? [2]

12. Explain in words what strings are matched by the following regular expression:

$$[a-zA-Z0-9._]+@[a-zA-Z0-9.-]+\.[a-zA-Z]{2,4}$$ [2]

14. (a) Alice, Bob, Charlie and Dianne each make the following statements:

   Alice: I am telling the truth.
   Bob: Alice is telling the truth.
   Charlie: Bob is telling the truth.
   Dianne: Charlie is lying.

   Only one of the 4 people is telling the truth. Which one? [1]

(b) They now make the following statements:

   Alice: Bob is lying.
   Bob: Charlie is lying.
   Charlie: I like beer.
   Dianne: 2+2=4.

   Now two of the four people are telling the truth. Which two? [1]

(c) They are now joined by Egbert. They each make the following statements:

   Alice: I like wine.
   Bob: Charlie is lying.
   Charlie: Alice is lying.
   Dianne: Alice likes beer.
   Egbert: Alice likes beer.

   Now three of the five people are telling the truth. Which ones? [1]
15. The four-digit number 2652 is such that any two consecutive digits from it make a multiple of 13. So, 26 = 13 \times 2, 65 = 13 \times 5, and 52 = 13 \times 4. Another number \( N \) has this same property, is 100 digits long, and begins in a 9. What is the last digit of \( N \)?

(A) 2  
(B) 3  
(C) 6  
(D) 9  

16. Name one international Computer Science journal

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**Programming knowledge**

Some of the following questions ask you to write small pieces of source code. You are free to use the programming language of your choice. Make reasonable assumptions about available library routines.

17. Consider the following fragment of assembly language. \( R1 \) and \( R2 \) are registers. The format of the instructions are “\textit{instruction operand1, operand2, result}”.

\[
\begin{align*}
\text{xor } R1, & \quad R2, \quad R2 \\
\text{xor } R1, & \quad R2, \quad R1 \\
\text{xor } R1, & \quad R2, \quad R2
\end{align*}
\]

What happens to the values originally stored in the registers?
18. Implement a routine that reads a list of integers from the user. The list is terminated when the user enters “-1”. The routine must disregard all negative integers and, for the remaining integers, display the average value of the integers (to two decimal digits), the minimum value, the maximum value, and the second largest value.

19. Implement a routine that takes, as its only parameter, a doubly-linked list of integers, sorts the values into a new doubly-linked list, and returns the new list.
20. Implement a recursive routine that accepts one parameter $x$ — an integer — and calculates the following function:

$$f(x) = \begin{cases} 
0 & \text{if } x = 0 \\
(f(x - 1) + x) & \text{if } x \text{ is odd} \\
(f(x - 1) + f(x - 2)) & \text{otherwise}
\end{cases}$$

21. Implement a non-recursive routine that calculates the same function as in Question 20.
22. Write a script or describe how you would employ command line tools to calculate the average file size in a given directory $D$. 

[5]