A: MATHEMATICS KNOWLEDGE

1. Calculate the following values exactly (i.e., no formulas) and write the answer down in decimal notation (base-10).

   (a) $2^9$ \hspace{1cm} [1]

   (b) The binary number $101011_2$ \hspace{1cm} [1]

   (c) $5! - 4!$ \hspace{1cm} [1]

   (d) $\binom{8}{6}$ \hspace{1cm} [1]

   (e) $100 + 101 + 102 + \cdots + 198 + 199 + 200$ \hspace{1cm} [1]

2. Calculate $f^{(n)}$, the $n$-th derivative of $f(x) = xe^x$. (In Leibniz’s notation: $f^{(n)} = \frac{d^n}{dx^n} xe^x$.) \hspace{1cm} [2]

3. Calculate $\int_0^1 \sqrt{x} \, dx$. \hspace{1cm} [2]
4. Calculate the following matrix product:

\[
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\begin{bmatrix}
2 & 0 & 1 \\
0 & 1 & 2
\end{bmatrix}
\]  

5. Twelve playing cards (such as those used in Bridge, Poker, and Blackjack) are shuffled and placed face-down on a table. There are five black cards and seven red cards, but, because they are face-down, you cannot see which is which. You are asked to pick three cards.

(a) Calculate the probability of picking two red cards and one black card.

(b) Calculate the probability that the first card you pick is black, the next card is red, and the last card is again black.

6. Consider the following sequence of numbers:

\[7 \ 2 \ 7 \ 2 \ 5 \ 2 \ 4 \ 3\]

(a) What is the mean value of the sequence?

(b) What is the median value of the sequence?

(c) What is the mode of the sequence?

(d) What is the standard deviation of the sequence?
7. Simplify (or rewrite) the expression

\[ (-a \implies \neg b) \implies \neg(a \implies b) \]

until you reach an expression of the form

\[ (W \land X) \lor (Y \land Z) \]

where \( W, X, Y, Z \in \{a, \neg a, b, \neg b\} \). Give the values of \( W, X, Y, Z \). [4]

B: COMPUTER SCIENCE KNOWLEDGE

8. Name three sorting algorithms and, for each algorithm, give its worst-case asymptotic time consumption. Example: dumbsort \( O(2^n) \) [6]

9. Explain or define each of the following concepts:

(a) Dynamic memory [1]

(b) Heap data structure [1]

(c) Object-oriented programming [1]

(d) SQL [1]

(e) HTTP [1]

(f) UML [1]
10. (a) What does the abbreviation “NP” (as in “P v. NP”) stand for?  
(b) What does it mean when we say that a problem is NP-complete?  
(c) Name one NP-complete problem.

11. (a) The structure of finite automata is often given as something like $M = (Q, \Sigma, \Delta, \hat{Q})$ or $M = (S, A, T, I)$. The exact notation is not important: explain in your own words what the four components are and their relationship to each other. 
(b) What is the difference between a DFA (deterministic finite automaton) and an NFA (nondeterministic finite automaton)?  
(c) What is the relationship between regular languages, DFAs, and NFAs?
12. Write down a regular expression describing the set of finite strings of zeros and ones in which every zero is followed by two consecutive ones. For instance, 10111011 is in this language, but 0011 is not. [2]


14. Name one international Computer Science journal [1]

15. Alice, Bob, and Carl all work for the same company. Alice is not married, but Carl is. Alice sends a message to Bob and Bob sends a message to Carl. Has an unmarried person sent a message to a married person? [2]

C: PROGRAMMING KNOWLEDGE

Some of the following questions ask you to write small pieces of source code. You are free to use the programming language of your choice. Make reasonable assumptions about available library routines.

16. What is the effect of the following piece of code? Under which circumstances will it fail?

    x := x * y
    y := x / y
    x := x / y

[3]

17. Write a script or describe how you would employ command line tools to count the number of files in the current working directory whose names contains the substring “ab”. [5]
18. Implement a routine that uses recursion to insert an integer into a linked list. The list may be empty, or may already contain a sequence of integers, stored in ascending order. The goal is to insert the new integer in the correct position. Your routine must accept two parameters: the integer \( x \) and the (singly-) linked, sorted list \( L \). The nodes of the linked list have the following definition:

```plaintext
Node {
    integer val;
    Node* next;
}
```

The routine must return the modified list when it is done.
19. Implement a routine that computes the same value as the function below, \textit{without} making use of recursion.

\begin{verbatim}
function f(integer k):
    if k <= 0:
        return 2
    else
        n = f(k - 1)
        return n * n
\end{verbatim}
20. We are given an array \( X[1\ldots N] \) of distinct integers that is \textit{almost-sorted} (in ascending order):

- For \( 1 < i < N \), the element in slot \( X[i] \) belongs in either slot \( X[i - 1] \) or slot \( X[i + 1] \);
- the element in \( X[1] \) belongs in either \( X[1] \) or \( X[2] \); and
- the element in \( X[N] \) belongs in either \( X[N - 1] \) or \( X[N] \).

In other words, when we compare the original array \( X \) to a sorted version of the same array, each element has moved at most one place from its starting position.

Implement a routine that accepts an almost-sorted integer array \( X \) as a parameter, and returns the minimum number of swaps required to sort the array. (Note, the routine should not actually sort the array, just calculate the minimum number of swaps.)